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Crack identification in beams using wavelet analysis

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Abstract

In this paper a simple method for crack identification in beam structures based on wavelet analysis is presented. The fundamental vibration mode of a cracked cantilever beam is analyzed using continuous wavelet transform and both the location and size of the crack are estimated. The position of the crack is located by the sudden change in the spatial variation of the transformed response. To estimate the size of the crack, an intensity factor is defined which relates the size of the crack to the coefficients of the wavelet transform. An intensity factor law is established which allows accurate prediction of crack size. The viability of the proposed method is investigated both analytically and experimentally in case of a cantilever beam containing a transverse surface crack. In the light of the results obtained, the advantages and limitations of the proposed method as well as suggestions for future work are presented and discussed.
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Keywords: Crack identification; Wavelet transform

1. Introduction

Cracks present a serious threat to the performance of structures since most of the structural failures are due to material fatigue. For this reason, methods allowing early detection and localization of cracks have been the subject of intensive investigation the last two decades. As a result, a variety of analytical, numerical and experimental investigations now exist. A review of the state of the art of vibration based methods for testing cracked structures has been published by Dimarogonas (1996).

A crack in a structure induces a local flexibility which affects the dynamic behaviour of the whole structure to a considerable degree. It results in reduction of natural frequencies and changes in mode shapes of vibration. An analysis of these changes makes it possible to identify cracks. In the pioneering work of Dimarogonas (1976) and Paipetis and Dimarogonas (1986) the crack was modelled as a local flexibility and the equivalent stiffness was computed using fracture mechanics methods. In that vein, Chondros and Dimarogonas (1980) developed methods to identify cracks in various structures relating the crack depth to the change in natural frequencies for known crack position. Adams and Cawley (1979) developed an experimental technique to estimate the location and depth of a crack from changes in natural frequencies.

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Gudmunson (1982) used a perturbation method to predict changes in natural frequencies of structures resulting from cracks, notches and other geometrical changes. Further work on crack identification via natural frequency changes was done by Anifantis et al. (1985). Using a similar approach Masoud et al. (1998) investigated the vibrational characteristics of a prestressed fixed–fixed beam with a symmetric crack and the coupling effect between crack depth and axial load. Narkis (1993) developed a closed form solution for the problem of a cracked beam, which he applied to study the inverse problem of localization of cracks on the basis of natural frequency measurements.

The main reason for the popularity of natural frequencies as damage indicators is that natural frequencies are rather easy to determine with a high degree of accuracy. A sensor placed on a structure and connected to a frequency analyzer gives estimates of several natural frequencies. Problems exist, however, when the size of the damage is small. The presence of measurement errors results in a degradation of the ability to predict the size of the crack accurately. The existing methods give a proper estimation of moderate cracks (about 20% of the height of the beam).

To overcome the aforementioned difficulties related to natural frequencies, many research studies have been focused on utilizing changes in mode shapes (Stubbs and Kim, 1966; Kim and Stubbs, 2002). The idea of using mode shapes as crack identification tool is the fact that the presence of a crack causes changes in the modal characteristics. Rizos et al. (1990) suggested a method for using measured amplitudes of two points of a cantilever beam vibrating at one of its natural modes to identify crack location and depth. Recently, an interesting comparison between a frequency—based and mode shape—based method for damage identification in beam like structures has been published by Kim et al. (2003). The advantage of using mode shapes is that changes in mode shapes are much more sensitive compared to changes in natural frequencies. Using mode shapes, however, has some drawbacks. The presence of damage may not significantly influence mode shapes of the lower modes usually measured. Furthermore, environmental noise and choice of sensors used can considerably affect the accuracy of the damage detection procedure. To overcome these difficulties, modal testing using scanning laser vibrometers have been developed (Stanbridge and Ewing, 1999). The laser vibrometer, used as a vibration transducer, has the advantage of being non-contacting and measures at a controlled position with high accuracy.

In the last few years, wavelet analysis has become a promising damage detection tool due to the fact that it is very accurate to detect localized abnormalities in a mode shape caused by the presence of a crack. It has useful localization characteristics and does not require the numerical differentiation of the measured data (Newland, 1994a,b). Wavelet transform can be implemented as fast as the Fourier transform and its main advantage is the fact that the local features in a signal can be identified with a desired resolution.

Deng and Wang (1998) applied the discrete wavelet transform to locate a crack along the length of a beam. Wang and Deng (1999) extended the analysis to a plate with a through-thickness crack. In the last study, the Haar wavelet were used with success. However, a method for estimating the crack extend has not been proposed. Haar wavelet were also used in the study of Quek et al. (2001). The authors were able to accurately detect relatively small cracks under both simply-supported and fixed-ended conditions. Here again, the estimation of the size of the crack is not discussed. Hong et al. (2002) used the Lipschitz exponent for the detection of singularities in beam modal data. The Mexican hat wavelet was used throughout the study and the crack size has been related to different values of the exponent. The correlation of the crack extent with the Lipschitz exponent is sensitive to both sampling distance and noise resulting in limited accuracy of the prediction.

In the present work, a method for crack identification in beam structures based on wavelet analysis is presented. The fundamental vibration mode of a cracked beam is wavelet transformed and both the location and size of the crack are estimated. For this purpose, a "symmetrical 4" wavelet having two vanishing moments is utilized. The position of the crack is located by the variation of the spatial signal at the site of the crack due to the high resolution property of the wavelet transform. To estimate the size of the crack, an intensity factor is defined which relates the size of the crack to the coefficients of the wavelet transform. An intensity factor law is established which allows accurate prediction of the crack size. The feasibility of the proposed method is investigated both analytically and experimentally in case of a cantilever beam containing a transverse surface crack. The influence of noise on the estimation of crack size has been also investigated. It is shown that noise added to mode shape increases the estimated crack size. In view of the results obtained, the limitations and the advantages of the proposed method as well as suggestions for future work are presented and discussed.

2. Wavelet transform background

A wavelet is a function with two important properties: oscillation and short duration. A function $\psi(x)$ is a wavelet if and only if its Fourier transform $\Psi(\omega)$ satisfies

$$\int_{-\infty}^{+\infty} \frac{|\Psi(\omega)|^2}{|\omega|^2} \mathrm{d}\omega < +\infty. \tag{1}$$

This condition implies that

$$\int_{-\infty}^{+\infty} \psi(u)\,\mathrm{d}u = 0, \tag{2}$$

which means that a wavelet is an oscillating function with zero mean value. For practical purposes it is also required the wavelet to be concentrated in a limited interval $[-K, K]$, or in other words have compact support.

The continuous wavelet transform of a function $f(x)$, where variable x is time or space, is defined as

$$Wf(u,s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(x)\psi^*\left(\frac{x-u}{s}\right) \mathrm{d}x, \tag{3}$$

where $\psi^*(x)$ is the complex conjugate of the wavelet function.

In translating Eq. (3) one might recognize the inner product of $f(x)$ with scaled and translated versions of the original wavelet function. Large values of scale s correspond to big wavelets and thus coarse features of $f(x)$, while low values of s correspond to small wavelets and fine details of $f(x)$.

An important property of the wavelet transform is its ability to react to subtle changes of the signal structure. To point this out, suppose that the wavelet used is the derivative of a continuous function $\phi(x)$ usually called the scaling function, i.e. $\psi(x) = \mathrm{d}\phi(x)/\mathrm{d}x$. The wavelet transform can be written

$$Wf(u,s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(x)\frac{\mathrm{d}}{\mathrm{d}u}\phi^*\left(\frac{x-u}{s}\right) \mathrm{d}x = \frac{\mathrm{d}}{\mathrm{d}u}\left\{\frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(x)\phi^*\left(\frac{x-u}{s}\right) \mathrm{d}x\right\}. \tag{4}$$

The continuous wavelet transform is proportional to the first derivative of $f(x)$ smoothed by the function $\phi(x)$. Of particular importance are the local maxima of $|Wf(u,s)|$ which, as explained above, are the local maxima of the derivative of $f(x)$ smoothed by $\phi(x)$. Mallat and Hwang (1992) connected the regularity of a function at a point $x = x_o$ with the decay of the local maxima of the wavelet modulus across scales. To detect singularities one has to examine the asymptotic decay of wavelet modulus maxima, as s goes to zero. If the coefficients decay at some rate as the scale decreases to zero, then x_o is a singular point of $f(x)$.

The regularity of a function at a point of time or space is characterized by its Hoelder exponent. More specifically, for an isolated singularity, i.e. non-oscillating singularity, the wavelet transform modulus maxima satisfy

$$|Wf(s,x)| \leqslant As^{h+1/2}, \tag{5}$$

where A is a constant and h is the Hoelder exponent. The Hoelder exponent gives information about the differentiability of a function more precisely. For example, if the value of the exponent is 1.5 at a point x_o, then we know that the function $f(x)$ is one time differentiable but not two times differentiable. The greater the value of the Hoelder exponent, the more regular is the function at this point. If an exponent h is assigned to an isolated singularity at point x_o then it is possible to interpolate all points of the function close to x_o with a curve of the form x^h. A practical way to calcutate the Hoelder exponent is obviously by rewriting Eq. (5) in the form

$$\log_2(|Wf(s,x)|) \leqslant \log_2(A) + (h + 1/2)\log_2(s). \tag{6}$$

By keeping the equality sign and plotting the coefficients on a logarithmic scale, A and h are calculated so as the error is minimized in the least square sense.

A question arises about which type of wavelet would be useful for the analysis. The answer is related to the concept of vanishing moments. A wavelet is said to have n vanishing moments if

$$\int_{-\infty}^{+\infty} x^n \psi(x)\,\mathrm{d}x = 0. \tag{7}$$

In simple terms, a wavelet with one vanishing moment does not "see" (is orthogonal to) linear functions. Two vanishing moments make a wavelet "blind" to quadratic functions as well. Using wavelets with more vanishing moments has the advantage of being able to measure the Hoelder regularity up to a higher order. On the other hand, as Daubechies (1992) proved, if a wavelet has n vanishing moments, its support length must be at least $2n - 1$. Localization worsens as support length increases and the same holds for the number of computations. Clearly, a compromise has to be made between the number of vanishing moments and support length. In most cases, the exponents to be calculated do not exceed 2 and so wavelets with two vanishing moments would suffice. In practice wavelets with higher number of vanishing moments give higher coefficients and more stable performance. After some experimentation, the authors considered the "symmetrical 4" wavelet with 4 vanishing moments a good candidate. As already mentioned, the wavelet transform provides information about the fourth derivative of the signal and permits additionally the investigation of the signal for all scales of interest. Another quite important issue is the role of the constant A entering Eq. (6). Constant A becomes the only parameter that describes the singularity if the exponent h has a fixed value. The Hoelder exponent describes the type of singularity. For example, a delta Dirac function is Hoelder −1 and a step function is Hoelder 1. If all singularities in a signal are of the same type, they should be characterized by the same exponent. If the magnitude of singularity changes, then only the constant A changes. In this sense, constant A might be considered as an intensity factor relating the depth of the crack to the coefficients of the wavelet transform.

3. Simulations on a cantilever beam

3.1. Vibration model of a cracked cantilever beam

Before applying the wavelet transform to experimental mode shapes numerical simulations were performed.

A cantilever beam of length ℓ, of uniform rectangular cross-section $w \times w$ with a crack located at ℓ_c is considered, as shown in Fig. 1(a). The crack is assumed to be open and have a uniform depth α.

Due to the localized crack effect, the beam can be simulated by two segments connected by a massless spring (Fig. 1(b)). For general loading, a local flexibility matrix relates displacements and forces. In our analysis, since only bending vibrations of thin beams are considered, the rotational spring constant is assumed to be dominant in the local flexibility matrix (Paipetis and Dimarogonas, 1986).

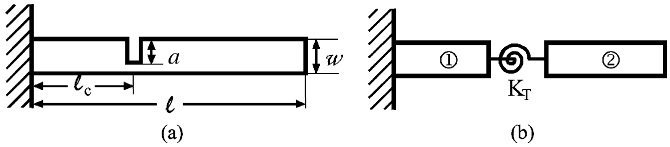

Fig. 1. (a) Cantilever beam under study, (b) cracked cantilever beam model.

The bending spring constant K_τ in the vicinity of the cracked section is given by

$$K_\tau = \frac{1}{C}, \quad C = (5.346w/EI)J(a/w), \tag{8}$$

where w is the depth of the beam, E is the modulus of elasticity of the beam, I is the area moment of inertia of the beam cross-section and $J(a/w)$ is the dimentionless local compliance function.

The function $J(a/w)$ has the form

$$J(a/w) = 1.8624(a/w)^2 - 3.95(a/w)^3 + 16.37(a/w)^4 - 37.226(a/w)^5 + 76.81(a/w)^6 - 126.9(a/w)^7 + 172(a/w)^8 - 43.97(a/w)^9 + 66.56(a/w)^{10}. \tag{9}$$

The displacement on each part of the beam is

$$\eta_1(x) = C_1 \cosh K_B x + C_2 \sinh K_B x + C_3 \cos K_B x + C_4 \sin K_B x,$$

$$\eta_2(x) = C_5 \cosh K_B x + C_6 \sinh K_B x + C_7 \cos K_B x + C_8 \sin K_B x, \tag{10}$$

with $K_B^4 = \omega^2 \rho A \ell / EI$. Here, A is the cross-section area, ω is the vibration angular frequency, ρ is the material density and C_i, $i = 1, 2, \ldots, 8$ are constants to be determined from the boundary conditions.

The boundary conditions at both ends are:

$$\begin{aligned} &\text{at } x = 0: \ \eta_1(0) = 0, \quad \eta_1'(0) = 0, \\ &\text{at } x = \ell: \ M_2(\ell) = 0, \quad F_2(\ell) = 0, \end{aligned} \tag{11}$$

where the prime denotes derivative with respect to x.

For the connection between the two segments conditions can be introduced, which impose continuity of displacement, bending moment and shear. Moreover, an additional condition imposes equilibrium between transmitted bending moment and rotation of the spring representing the crack. Consequently, the boundary conditions at the crack can be expressed as follows:

$$\eta_1(\ell_c) = \eta_2(\ell_c), \quad M_1(\ell_c) = M_2(\ell_c), \quad F_1(\ell_c) = F_2(\ell_c),$$

$$-EI \frac{\partial^2}{\partial x^2} \eta_1(\ell_c) = K_\tau \left[\frac{\partial}{\partial x} \eta_1(\ell_c) - \frac{\partial}{\partial x} \eta_2(\ell_c) \right]. \tag{12}$$

The resulting characteristic equation for the above described system can be solved numerically and both the natural frequencies and mode shapes of the beam can be obtained.

For numerical simulations a plexiglas cantilever beam of total length 300 mm and rectangular cross-section 20×20 mm^2 is considered. A crack of relative depth 20% is introduced at $x = 60$ mm from the clamped end. Using the above described procedure, the fundamental vibration mode of the beam was calculated. The results are shown in Fig. 2. Displacement data follow a sampling distance of 1 mm resulting in a number of 301 points available. The data are normalized so that the maximum displacement value equals one.

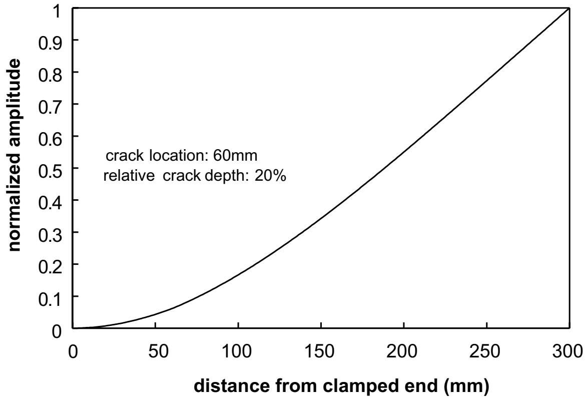

Fig. 2. Calculated fundamental vibration mode of the cracked cantilever beam.

3.2. Determination of crack location

To determine the location of the crack the response data in Fig. 2 were wavelet transformed. The continuous wavelet transform is preferred instead of the discrete version, as the redundancy of information it provides is useful for analysis purposes. The wavelet transform is implemented for scales 1–25 with the "symmetrical 4" as the analyzing wavelet. The available resolution limits the analysis to a minimum scale of one. On the other hand, for scales of about 25 the singularity induced by the presence of the crack cannot be considered as isolated.

The results of the wavelet analysis are presented in Fig. 3 for scales 2, 5, 10 and 15. It is obvious that the wavelet transform coefficients exhibit a maximum at $x = 60$ mm. This implies the presence of a singularity. To be certain about the presence of a crack, however, one has to examine the trend of wavelet maxima at this point as the scale decreases. The results are shown in Fig. 4. It can be clearly seen that the absolute value of the wavelet maxima decreases in a regular manner with decreasing scale. Ideally, it should tend to zero for zero scale, but as already mentioned, one is forced to draw conclusions from the trend of coefficients to the minimum scale of 1.

The absence of coefficients of significant value in Fig. 4 at any other location away from the crack is characteristic. This is attributed to the fact that the analyzed data stem from theoretical computations of the response and hence contain no noise or measurement errors. In a real experiment, however, noise is expected to corrupt the response data. It is known (Angrisani et al., 1999) that the wavelet transform coefficients behave in a completely different manner if they are generated by noise disturbances. They characterized by negative Hoelder exponents and consequently, the wavelet modulus maxima increase with decreasing scale. This fact provides a way of discriminating singular points generated by noise. This issue will be examined in detail in a subsequent section.

3.3. Estimation of crack depth

Finding the location of the crack is only half the way, the other half being the estimation of crack depth. It has been already mentioned that constant A is the only parameter that changes if the Hoelder exponent, which describes the type of the singularity, has a fixed value. Therefore, constant A has been defined as intensity factor since its magnitude characterizes the size of the crack.

Fig. 5 is a plot of the wavelet maxima coefficients versus scale for crack depths varying from 10% to 80%. The Hoelder exponent has a constant value of 1.005 for all cases. This means that the mode function is just

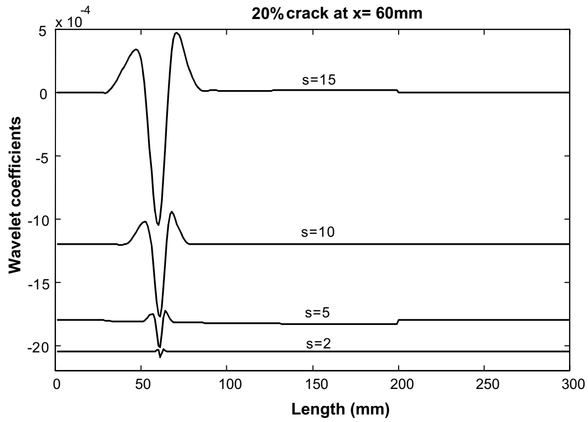

Fig. 3. Wavelet analysis of different scales based on the calculated displacement response of the cracked cantilever beam (20% crack at $x = 60$ mm from clamped end).

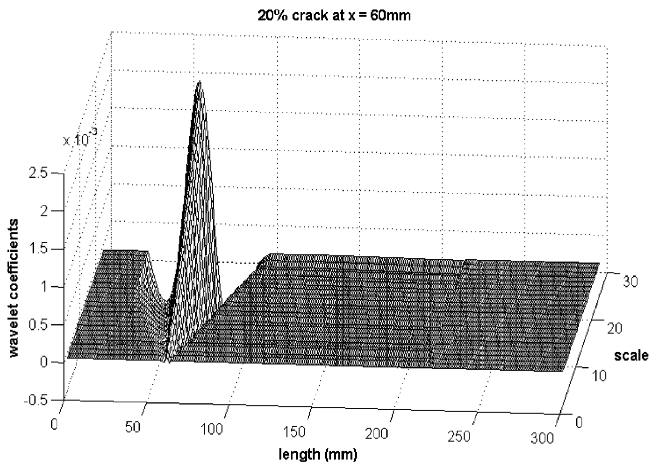

Fig. 4. Three-dimensional plot of the wavelet transform showing the trend of the wavelet modulus maxima at crack location.

on time differentiable at the point where the crack is introduced. In other words, the amplitude of the mode increases locally in an almost linear fashion because of the presence of the crack. One has a set of parallel lines (fixed Hoelder exponent) which can be identified from their different constant A, i.e. different intensity factors. The intensity factor increases with crack depth according to a second order polynomial law as shown in Fig. 6. This enables an estimation of crack depth. For that purpose, the intensity factor A is calculated and using Fig. 6 the crack depth can be estimated.

There are two important issues, however, to be noticed. The intensity factor depends to some extent on the analyzing wavelet and on the location of the crack. The property of a wavelet that affects the intensity factor is the number of vanishing moments. This problem is resolved by using the same wavelet throughout the investigation. To examine the influence of crack location on the intensity factor Fig. 6, presenting the cases of different crack locations at $x = 60$ and 100 mm, is considered. It can be seen that the values of the intensity factor for a crack located at $x = 100$ mm are lower compared to the corresponding values for

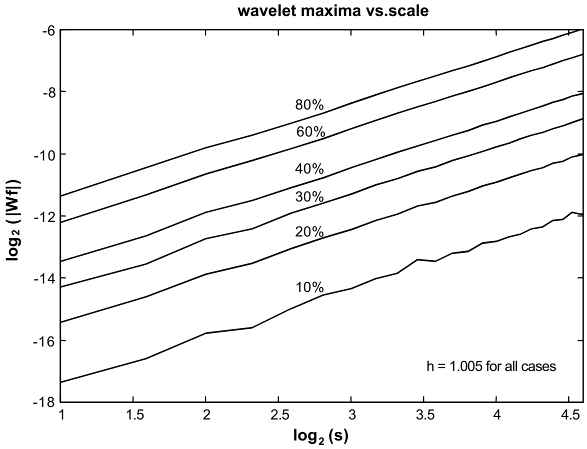

Fig. 5. Wavelet maxima coefficients versus scales for different crack depths (crack location $x = 60$ mm from clamped end).

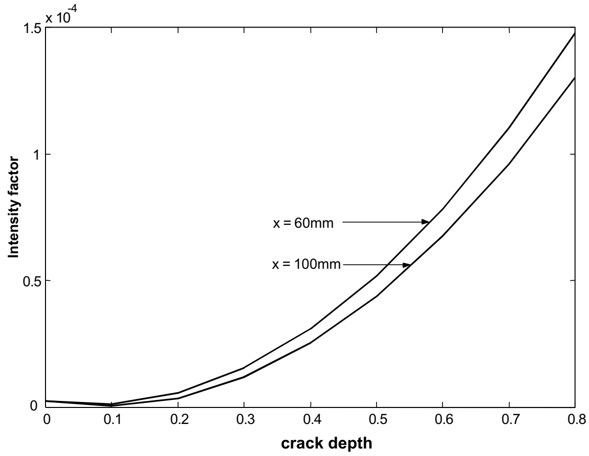

Fig. 6. Intensity factor versus normalized crack depth for different crack position.

crack location at $x = 60$ mm. This is due to the fact that the wavelet transform coefficients are proportional to the derivatives of the response signal. At $x = 100$ mm, the slope of the fundamental mode is 1.49 times higher than the slope at $x = 60$ mm. This in turn means that the corresponding wavelet coefficients must be 1.49 times lower.

4. Effect of noise on wavelet analysis

To investigate the effect of noise or measurement errors on the proposed detection process, we consider the theoretical response data in Fig. 2 and add some noise so that the mean error reaches a certain value. In our study a mean error of 1% was introduced. Fig. 7 shows the exact displacement response, predicted by the theoretical model, along with the corrupted data. The results correspond to a cantilever beam of length

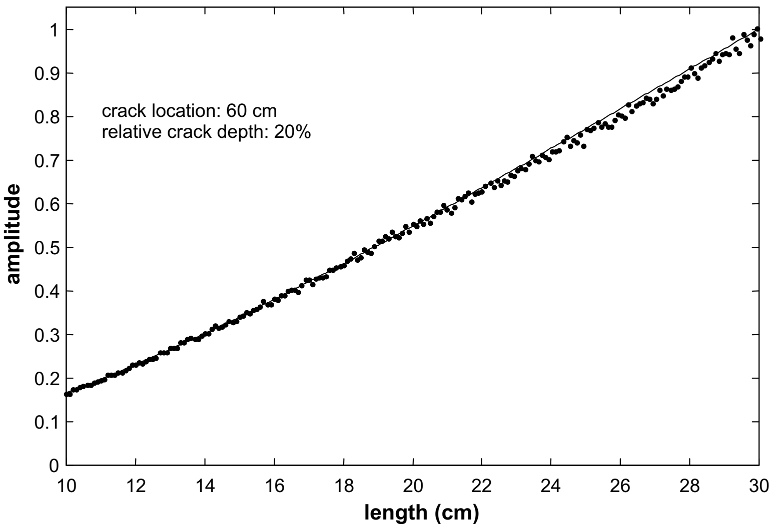

Fig. 7. Simulated noisy fundamental vibration mode of the cracked cantilever beam with mean error 1%.

300 mm of rectangular cross-section 20×20 mm^2 with a crack of relative depth 30% located at $x = 60$ mm from the clamped end.

The noisy response data were wavelet analyzed and the results are presented in Fig. 8. The wavelet coefficients exhibit local maxima at various locations along the beam. It should be noticed, however, that the coefficients do not decrease with scale but they increase or remain at a constant level. This behaviour implies that all these singularities are generated by noise disturbances. On the other hand, there is a certain location at $x = 60$ mm where the wavelet transform maxima decreases regularly with scale. This behaviour implies that the singularity at $x = 60$ mm is due to the presence of a crack.

Keeping the crack location fixed, several cases with varying crack depth have been investigated. It follows, that crack localization is easier in case of large cracks (greater than 40%) and somewhat obscured for small cracks (less than 20%). In all cases, however, the location of a crack can be accurately determined.

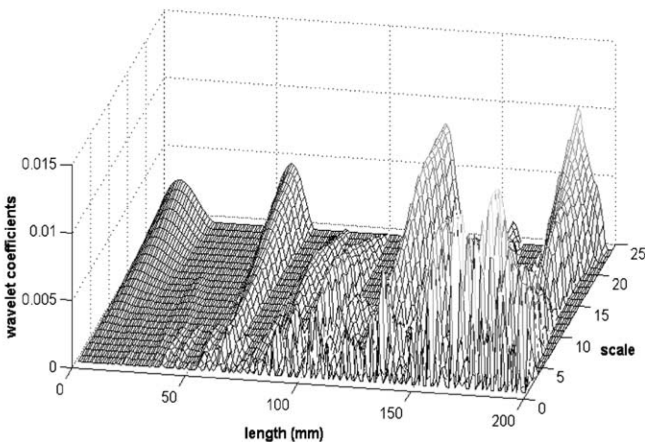

Fig. 8. Three-dimensional plot of the wavelet transform of the noisy displacement data.

Using the noisy data, the behaviour of the Hoelder exponent has been also investigated. It turns out that the Hoelder exponents can be accurately estimated for crack depths greater than 50%. For moderate cracks the Hoelder exponents are lower compared to the exact value 1.005. The calculated values decrease with decreasing crack depth reaching $h = 0.76$ in case of a 10% crack. This can be easily explained by recalling that transients and uncorrelated noise are characterized by negative Hoelder exponents. Noise influences more small wavelet maxima coefficients which correspond to small cracks. An important point to be noticed is that lower scales should be ignored in the interpolation procedure as they are corrupted by noise. The Hoelder exponent is thus calculated for scales where the wavelet maxima coefficients show a more or less linear behaviour.

The values of constant A, i.e. intensity factor, continue to follow a second order polynomial law, but appear to be twice as high in magnitude compared to the values obtained by analyzing the exact data. This means that noise increases the apparent magnitude of the singularity, i.e. the size of the crack. Therefore, an accurate estimation of crack depth seems problematic. If the average error added to the displacement data is further increased, say about 5%, both localization and size prediction become practically impossible.

5. Experimental investigation

To validate the analytical results of the wavelet analysis, an experiment on a plexiglas beam has been performed. A 300 mm plexiglas cantilever beam of cross-section 20×20 mm^2 was clamped at a vibrating table. An electromagnetic vibrator by Link and two B&K accelerometers were utilized. Harmonic excitation was used via a 2110 B&K analyzer and the fundamental mode of vibration was investigated.

The vibration amplitude was measured with a sampling distance of 7.5 mm, which was the effective diameter of the accelerometer used, so that a total number of 39 measuring points were obtained.

Mode shapes were measured by using two calibrated accelerometers mounted on the beam. One accelerometer was kept at the clamped end as the reference input, while the second accelerometer was moved along the beam to measure the mode amplitude. A plot of the measured fundamental mode shape of the beam with a crack of relative depth 30% located at $x = 60$ mm from the clamped end is shown in Fig. 9. Because of the spare sampling, the wavelet transform if implemented directly would detect many points of the sampled data as singularities. Therefore, to smooth the transition from one point to another an oversampling procedure is necessary. For that purpose, a cubic spline interpolation was used which resulted

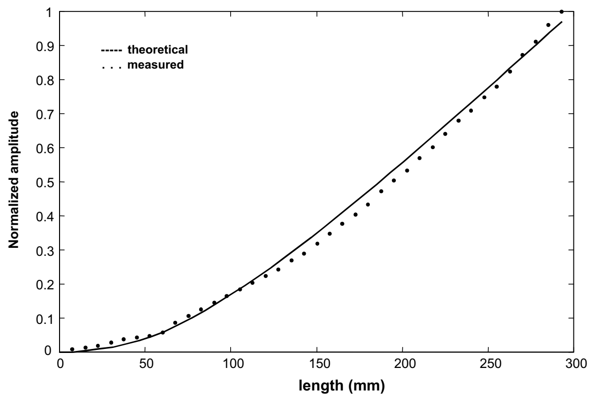

Fig. 9. Comparison between calculated and measured fundamental vibration mode of the cracked cantilever beam (30% crack located at $x = 60$ mm from clamped end.

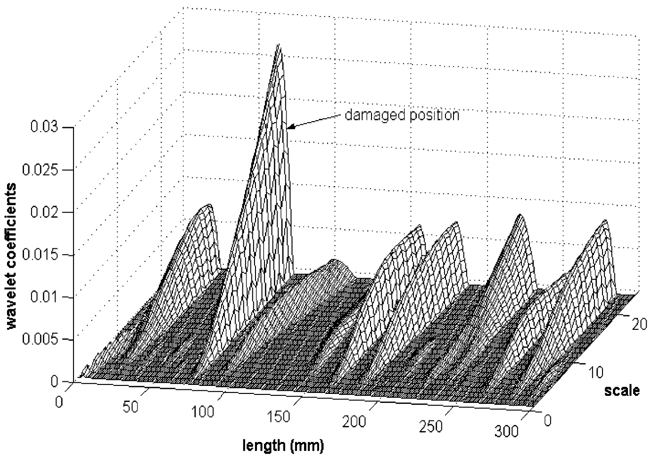

Fig. 10. Three-dimensional wavelet transform of the experimental mode shape of the cracked cantilever beam.

to a total of 390 points available. Fig. 10 presents a three-dimensional plot of the wavelet transform of the experimental data. The analysis was carried out for scales 1–25 in accordance with the analysis of the theoretical data. It can be seen that the wavelet transform coefficients have significant values in more than one points along the beam. A more careful observation, however, reveals that the coefficients do not decrease regularly with scale. This observation provides a way to discriminate real singular points from singularities generated by noise. On the other hand, at $x = 60$ mm, where the crack is located, the coefficients exhibit higher values compared to all other points and more important, they decrease in a regular manner with scale. Therefore, at $x = 60$ mm a singularity generated by a crack is present.

To improve the results of the analysis a denoising algorithm was implemented, so that only coefficients of absolute value more than 50% of the maximum value of each scale are considered. In other words, a threshold equal to 0.5 of the maximum value has been utilized. The results are shown in Fig. 11. It follows clearly that the crack is located at $x = 60$ mm.

To estimate the size of the crack, the wavelet maxima for different scales were calculated at crack location. The results are shown in Fig. 12. The resulting values of the Hoelder exponent and intensity factor

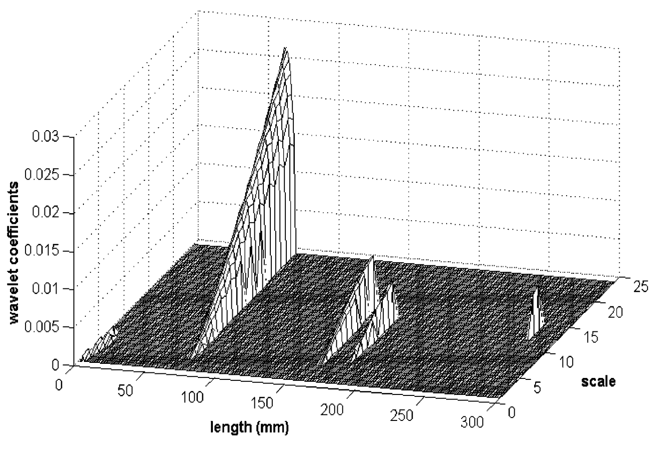

Fig. 11. Three-dimensional plot of the wavelet transform maxima versus scale for the denoised measured mode shape.

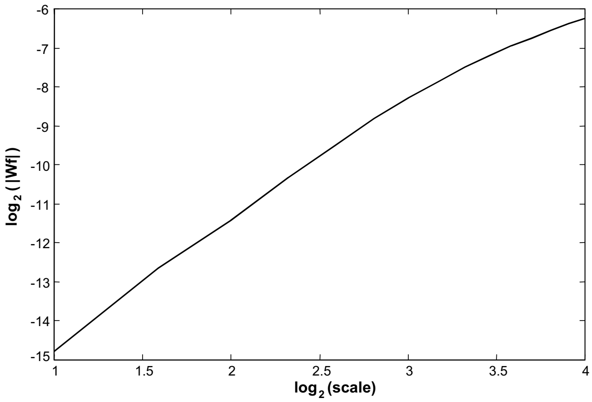

Fig. 12. Wavelet maxima versus scale.

are $h = 1.24$ and $A = 10.35 \times 10^{-5}$ respectively. The value of the Hoelder exponent is 20% higher compared to the theoretical value. This means that the singularity generated by the presence of the crack appears to be smoother as a result of inadequate sampling density. The prediction scheme based on the polynomial law yields a crack depth of approximately 50% while the actual crack depth is 30%. This is expected since the prediction scheme is based on the exact Hoelder exponent equal to $h = 1.005$. As the exponent increases the intensity factor also increases resulting in a larger singularity, i.e. a larger crack.

6. Conclusion

A method for crack identification in beam structures based on wavelet analysis has been presented. For that purpose, a cracked cantilever beam having a transverse surface crack has ben investigated both analytically and experimentally using wavelet transform.

The location of the crack is determined by the variation of the spatial response signal at the site of the crack. Such local variations usually do not appear from the measured data they are, however, discernible as singularities when using wavelet analysis due to its high resolution properties.

For the characterization of the crack, i.e. estimation of crack depth, the notion of the intensity factor were utilized. The intensity factor relates the size of the crack to the coefficients of the wavelet transform and can be easily estimated by examining the coefficient maxima at the vicinity of the crack. An intensity factor law has been established which allows accurate estimation of crack depth.

Before applying the method to measured data, the sensitivity to random signal noise has been investigated. Using controlled simulation examples, a criterion based on the behaviour of the coefficients has been established for distinguishing between singularities generated by noise from those caused by the presence of a crack. Therefore, wavelet analysis can be used for accurate crack localization utilizing measured data. The investigation revealed that small errors (of the order of 2%) in the displacement response give intensity factor values increased by a factor of two. Hence, a reliable estimation of crack depth becomes questionable in case of small cracks.

The numerical results were confirmed by the application of wavelet analysis to actual experimental mode shapes of a cracked cantilever beam. Using the noisy experimental data, the location of a crack was accurately determined by making use of the different behaviour of the wavelet coefficients. To improve the prediction accuracy a denoising algorithm was implemented excluding singularities caused by noise.

The estimation of crack depth based on experimental data provided difficulties. The measurement errors result in higher values of the intensity factor which in turn leads to overestimation of the crack depth.

In conclusion, the presented results provide a foundation for using wavelet analysis as efficient crack detection tool. The advantage of using wavelet analysis is that perturbations in the structural response caused by the presence of a crack can be detected with the desired resolution. Further work is needed to investigate the sensitivity of wavelet analysis to random noise of measured data. It seems that a key issue is the spatial resolution and accuracy of the used response signal. Therefore, new sensors or measuring techniques able to pick-up the perturbations caused by the presence of a crack would enhance the potential of the method. In that vein, laser vibrometers could be proved to be a useful tool for non-contacting and fast measurement of mode shapes with the desired accuracy.

The results of the present work refer to a cantilever beam but can be easily extended to include more complex structures and boundary conditions. Work is already under way to explore the application of the proposed prediction technique to more complicated structures. These include multicracked beams and cracked plates.

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